FAKULTET ZAŠTITE NA RADU

REŠENJA ZADATAKA SA PRIJEMNOG ISPITA IZ MATEMATIKE

U Nišu 06.09.2016.god.

1. Uprostiti izraz

$$\frac{1+x^{-2}}{1-x^{-2}} \cdot \left(\frac{2x-1}{x}-1\right) =?$$

Rešenje.

$$\frac{1+x^{-2}}{1-x^{-2}} \cdot \left(\frac{2x-1}{x}-1\right) = \frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}} \cdot \frac{2x-1-x}{x} = \frac{\frac{x^2+1}{x^2}}{\frac{x^2-1}{x^2}} \cdot \frac{x-1}{x} =$$

$$\frac{x^2+1}{x^2-1} \cdot \frac{x-1}{x} = \frac{x^2+1}{(x-1)(x+1)} \cdot \frac{x-1}{x} = \frac{x^2+1}{x+1} \cdot \frac{1}{x} = \frac{x^2+1}{x(x+1)}.$$

2. Rešiti sistem jednačina

$x-y+1=0$

$x^2+xy+2y^2-3x+y-10=0.$

Rešenje.

$x-y+1=0$

$\underline{x^2+xy+2y^2-3x+y-10=0}$

$x=y-1$

$\underline{x^2+xy+2y^2-3x+y-10=0}$

$x=y-1$

$\underline{(y-1)^2+(y-1)y+2y^2-3(y-1)+y-10=0}$

$x=y-1$

$\underline{y^2-2y+1+y^2-y+2y^2-3y+3+y-10=0}$

$x=y-1$

$\underline{4y^2-5y-6=0}$

$x=y-1$

$\underline{y_{1,2}=\dfrac{5\pm\sqrt{25+96}}{8}=\dfrac{5\pm 11}{8}}$

$x=y-1$

$\underline{y_1=2, \qquad y_2=-\dfrac{3}{4}}$

$x_1=y_1-1=2-1=1, \qquad x_2=y_2-1=-\dfrac{3}{4}-1=-\dfrac{7}{4}$

Rešenja sistema su $(1,2)$ i $\left(-\dfrac{7}{4},-\dfrac{3}{4}\right)$.

3. Izračunati
$$(1+3i)^2+(3+4i)\cdot(2-3i)+\frac{1+i}{1-i}=?$$

Rešenje.

$(1+3i)^2+(3+4i)\cdot(2-3i)+\dfrac{1+i}{1-i}=$

$1+6i+9i^2+6-9i+8i-12i^2+\dfrac{(1+i)(1+i)}{(1-i)(1+i)}=$

$1+6i-9+6-i+12+\dfrac{1+2i+i^2}{1-i^2}=5i+10+\dfrac{1+2i-1}{1+1}=5i+10+i=10+6i.$

4. Ako je $\sin\alpha=\dfrac{7}{25}$ i $\alpha\in\left(\dfrac{\pi}{2},\pi\right)$, izračunati $\cos\alpha$, $\operatorname{tg}\alpha$ i $\sin 2\alpha$.

Rešenje. Kako je $\sin\alpha=\dfrac{7}{25}$ i $\alpha\in\left(\dfrac{\pi}{2},\pi\right)$, to je $\cos\alpha<0$, pa je

$\cos\alpha=-\sqrt{1-\sin^2\alpha}=-\sqrt{1-\left(\dfrac{7}{25}\right)^2}=-\sqrt{1-\dfrac{49}{625}}=-\sqrt{\dfrac{576}{625}}=-\dfrac{24}{25},$

$\operatorname{tg}\alpha=\dfrac{\sin\alpha}{\cos\alpha}=\dfrac{\frac{7}{25}}{-\frac{24}{25}}=-\dfrac{7}{24},\quad \sin 2\alpha=2\sin\alpha\cos\alpha=2\cdot\dfrac{7}{25}\cdot\left(-\dfrac{24}{25}\right)=-\dfrac{336}{625}.$

5. Rešiti jednačinu po x: $\log_5 x+\log_{25}x=\log_{\frac{1}{5}}\sqrt{3}.$

Rešenje. $\log_5 x+\log_{25}x=\log_{\frac{1}{5}}\sqrt{3}\,,\qquad x>0$

$\log_5 x+\log_{5^2}x=\log_{5^{-1}}\sqrt{3}$

$\log_5 x+\dfrac{1}{2}\log_5 x=-\log_5\sqrt{3}\quad /\cdot 2$

$2\log_5 x+\log_5 x+2\log_5\sqrt{3}=0$

$3\log_5 x+\log_5(\sqrt{3})^2=0$

$\log_5 x^3+\log_5 3=0$

$\log_5 3x^3=0$

$3x^3=1$

$x^3=\dfrac{1}{3}$

$x=\left(\dfrac{1}{3}\right)^{\frac{1}{3}}$

Dobijena vrednost nepoznate x predstavlja rešenje date jednačine jer zadovoljava uslov $x>0$.