FAKULTET ZAŠTITE NA RADU

REŠENJA ZADATAKA SA PRIJEMNOG ISPITA IZ MATEMATIKE

Niš, 28.6.2018.

1. Uprostiti izraz

$$\left(1-\frac{3x^2}{1-x^2}\right):\left(\frac{x}{x-1}+1\right)=?$$

Rešenje.

$$\left(1-\frac{3x^2}{1-x^2}\right):\left(\frac{x}{x-1}+1\right)=\frac{1-x^2-3x^2}{1-x^2}:\frac{x+x-1}{x-1}=$$
$$\frac{1-4x^2}{1-x^2}\cdot\frac{x-1}{2x-1}=\frac{(1-2x)(1+2x)}{(1-x)(1+x)}\cdot\frac{1-x}{1-2x}=\frac{1+2x}{1+x}.$$

2. Rešiti sistem jednačina

$$\begin{aligned}&x-y=4\\&\frac{3x-2}{y+5}+\frac{y}{x}=2.\end{aligned}$$

Rešenje.

$$\begin{aligned}&x-y=4\\&\underline{\frac{3x-2}{y+5}+\frac{y}{x}=2}\\&y=x-4\\&\underline{\frac{3x-2}{x+1}+\frac{x-4}{x}=2\ /\cdot x(x+1)}\\&y=x-4\\&\underline{x(3x-2)+(x+1)(x-4)=2x(x+1)}\\&y=x-4\\&\underline{3x^2-2x+x^2-4x+x-4=2x^2+2x}\\&y=x-4\\&\underline{2x^2-7x-4=0}\end{aligned}$$

$y = x - 4$

$x_{1,2} = \dfrac{7 \pm \sqrt{49+32}}{4} = \dfrac{7 \pm 9}{4}$

$y = x - 4$

$x_1 = 4, \qquad x_2 = -\dfrac{1}{2}$

$y_1 = x_1 - 4 = 4 - 4 = 0, \qquad y_2 = x_2 - 4 = -\dfrac{1}{2} - 4 = -\dfrac{9}{2}$

Rešenja sistema su $(4, 0)$ i $(-\dfrac{1}{2}, -\dfrac{9}{2})$.

3. Izračunati

$$(1+2i)^2 + (3-4i)\cdot(2+i) + \frac{3+i}{3-i} = ?$$

Rešenje.

$(1+2i)^2 + (3-4i)\cdot(2+i) + \dfrac{3+i}{3-i} =$

$1 + 4i + 4i^2 + 6 + 3i - 8i - 4i^2 + \dfrac{(3+i)(3+i)}{(3-i)(3+i)} =$

$7 + 4i - 4 - 5i + 4 + \dfrac{9 + 6i + i^2}{9 - i^2} = 7 - i + \dfrac{9 + 6i - 1}{9 + 1} = 7 - i + \dfrac{8 + 6i}{10} =$

$7 - i + \dfrac{4 + 3i}{5} = \dfrac{35 - 5i + 4 + 3i}{5} = \dfrac{39 - 2i}{5}.$

4. Ako je $\cos\alpha = \dfrac{15}{17}$ i α oštar ugao, izračunati $\operatorname{ctg}\left(\dfrac{\pi}{2} - \alpha\right)$.

Rešenje.

Kako je $\cos\alpha = \dfrac{15}{17}$ i α oštar ugao, to je $\sin\alpha > 0$, pa je

$\sin\alpha = \sqrt{1 - \cos^2\alpha} = \sqrt{1 - \left(\dfrac{15}{17}\right)^2} = \sqrt{1 - \dfrac{225}{289}} = \sqrt{\dfrac{64}{289}} = \dfrac{8}{17}$

$\operatorname{ctg}\left(\dfrac{\pi}{2} - \alpha\right) = \operatorname{tg}\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\frac{8}{17}}{\frac{15}{17}} = \dfrac{8}{15}.$

5. Rešiti jednačinu po x: $\dfrac{2}{\log x + 1} = 1 - \dfrac{1}{5 - \log x}$.

Rešenje.

$$\frac{2}{\log x + 1} = 1 - \frac{1}{5 - \log x}\,, \qquad x > 0 \wedge \log x + 1 \neq 0 \wedge 5 - \log x \neq 0$$

$$x > 0 \wedge x \neq \frac{1}{10} \wedge x \neq 10^5$$

Smena: $\log x = t$

$$\frac{2}{t+1} = 1 - \frac{1}{5-t} \quad /(t+1)(5-t)$$

$$2(5-t) = (t+1)(5-t) - (t+1)$$

$$10 - 2t = 5t - t^2 + 5 - t - t - 1$$

$$10 - 2t = 3t - t^2 + 4$$

$$t^2 - 5t + 6 = 0$$

$$t_{1,2} = \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm 1}{2}$$

$$t_1 = 3, \qquad t_2 = 2$$

$$(\log x)_1 = 3, \qquad (\log x)_2 = 2$$

$$x_1 = 10^3, \qquad x_2 = 10^2$$

$$x_1 = 1000, \qquad x_2 = 100$$

Dobijene vrednosti nepoznate x predstavljaju rešenja date jednačine jer zadovoljavaju uslove pod kojima ona postoji.