FAKULTET ZAŠTITE NA RADU

REŠENJA ZADATAKA SA PRIJEMNOG ISPITA IZ MATEMATIKE

Niš, 5.9.2017.

1. Uprostiti izraz

$$\left(\frac{6a}{a^2-4}+\frac{a+1}{a+2}-\frac{2a-1}{a-2}\right):\left(1-\frac{a^2+a-4}{a^2-4}\right)=?$$

Rešenje.

$$\left(\frac{6a}{a^2-4}+\frac{a+1}{a+2}-\frac{2a-1}{a-2}\right):\left(1-\frac{a^2+a-4}{a^2-4}\right)=$$
$$\frac{6a+(a+1)(a-2)-(2a-1)(a+2)}{(a-2)(a+2)}:\frac{(a^2-4)-(a^2+a-4)}{a^2-4}=$$
$$\frac{6a+a^2-2a+a-2-2a^2-4a+a+2}{a^2-4}\cdot\frac{a^2-4}{a^2-4-a^2-a+4}=$$
$$\frac{-a^2+2a}{1}\cdot\frac{1}{-a}=\frac{a^2-2a}{a}=\frac{a(a-2)}{a}=a-2.$$

2. Rešiti sistem jednačina

$x+y=6$

$x^2+y^2=2\,(xy+2).$

Rešenje.

$x+y=6$

$\underline{x^2+y^2=2\,(xy+2)}$

$y=6-x$

$\underline{x^2+(6-x)^2=2\,(x(6-x)+2)}$

$y=6-x$

$\underline{x^2+36-12x+x^2=2\,(6x-x^2+2)}$

$y=6-x$

$\underline{2x^2-12x+36=-2x^2+12x+4}$

$y=6-x$

$\underline{4x^2-24x+32=0}\,/:4$

$y=6-x$

$\underline{x^2-6x+8=0}$

$y = 6 - x$
$x_{1,2} = \dfrac{6 \pm \sqrt{36-32}}{2} = \dfrac{6 \pm 2}{2}$

$y = 6 - x$
$x_1 = 4, \qquad x_2 = 2$

$y_1 = 6 - x_1 = 6 - 4 = 2, \qquad y_2 = 6 - x_2 = 6 - 2 = 4$

Rešenja sistema su $(4,2)$ i $(2,4)$.

3. Izračunati

$$(2-3i)\cdot(3+4i) + \frac{1-i}{1+i} + (2+i)^2 + (1+i)^4 = ?$$

Rešenje.

$(2-3i)\cdot(3+4i) + \dfrac{1-i}{1+i} + (2+i)^2 + (1+i)^4 =$

$6 + 8i - 9i - 12i^2 + \dfrac{1-i}{1+i}\cdot\dfrac{1-i}{1-i} + 4 + 4i + i^2 + (1+2i+i^2)^2 =$

$6 - i + 12 + \dfrac{1-2i+i^2}{1-i^2} + 4 + 4i - 1 + (1+2i-1)^2 =$

$21 + 3i + \dfrac{1-2i-1}{1+1} + 4i^2 = 21 + 3i - i - 4 = 17 + 2i.$

4. Izračunati

$$\cos\left(\frac{\pi}{3} - \alpha\right) = ?$$

ako je $\alpha \in \left(\dfrac{3\pi}{2}, 2\pi\right)$ i $\cos\alpha = \dfrac{2}{5}$.

Rešenje.

Kako je $\cos\alpha = \dfrac{2}{5}$ i $\sin\alpha < 0$ jer je $\alpha \in \left(\dfrac{3\pi}{2}, 2\pi\right)$, to je

$\sin\alpha = -\sqrt{1-\cos^2\alpha} = -\sqrt{1-\left(\dfrac{2}{5}\right)^2} = -\sqrt{1-\dfrac{4}{25}} = -\sqrt{\dfrac{21}{25}} = -\dfrac{\sqrt{21}}{5}$,
pa je:

$\cos\left(\dfrac{\pi}{3}-\alpha\right) = \cos\dfrac{\pi}{3}\cdot\cos\alpha + \sin\dfrac{\pi}{3}\cdot\sin\alpha = \dfrac{1}{2}\cdot\dfrac{2}{5} + \dfrac{\sqrt{3}}{2}\cdot\dfrac{-\sqrt{21}}{5} = \dfrac{2-\sqrt{63}}{10} =$

$\dfrac{2-3\sqrt{7}}{10}$.

5. Rešiti jednačinu po x

$$\log_7 2 + \log_{49} x = \log_{\frac{1}{7}} \sqrt{3}.$$

Rešenje.

$\log_7 2 + \log_{49} x = \log_{\frac{1}{7}} \sqrt{3}\,, \qquad x > 0$

$\log_7 2 + \log_{7^2} x = \log_{7^{-1}} \sqrt{3}$

$\log_7 2 + \dfrac{1}{2} \log_7 x = -\log_7 \sqrt{3} \quad /\cdot 2$

$2\log_7 2 + \log_7 x + 2\log_7 \sqrt{3} = 0$

$\log_7 2^2 + \log_7 x + \log_7 (\sqrt{3})^2 = 0$

$\log_7 4 + \log_7 x + \log_7 3 = 0$

$\log_7 (4 \cdot x \cdot 3) = 0$

$12x = 7^0$

$12x = 1$

$x = \dfrac{1}{12}$

Dobijena vrednost nepoznate x predstavlja rešenje date jednačine jer zadovoljava uslov $x > 0$.